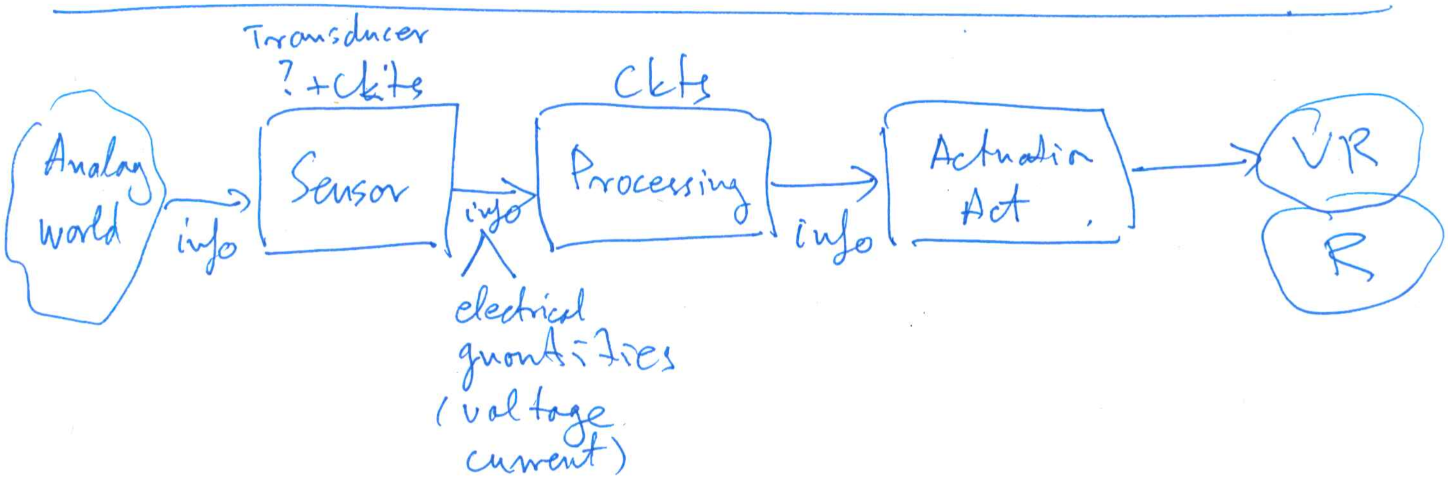
# Lecture 1 - Module 2

Today:

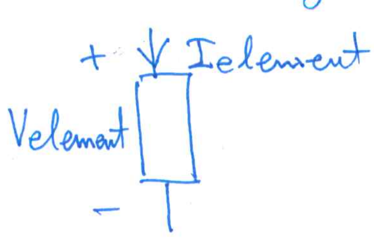
* Sensing, Processing & Actuation Systems
* Circuit Analysis Algorithm (Note 11)




## Electrical Circuit Analysis Algorithm (tool) | SPICE

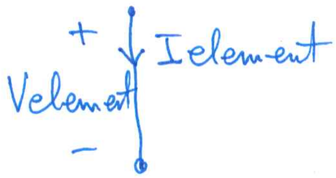
| Quantities: | Analytical Symbol | Units |
|---|---|---|
| Voltage | V | Volts [V] |
| Current | I | Amps [A] |
| Resistance | R | Ohms [Ω] |

Circuit diagram (collection of elements, where each element has some voltage across it and some current through it)



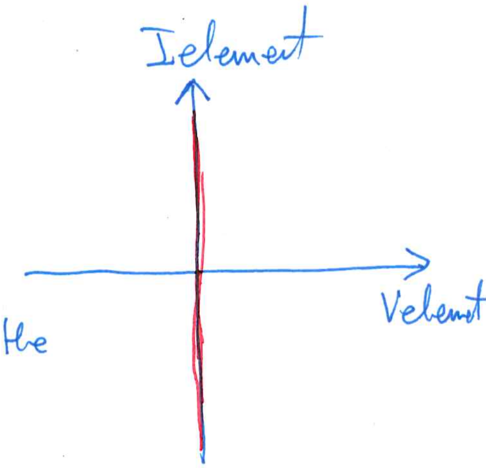
# Key circuit elements:

## 1. Wire

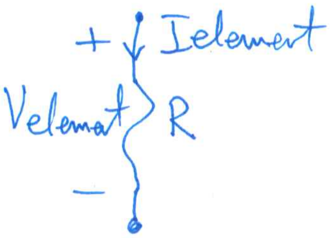



$V_{element} = 0$

$I_{element} = ?$
(set by the ext. circuit)

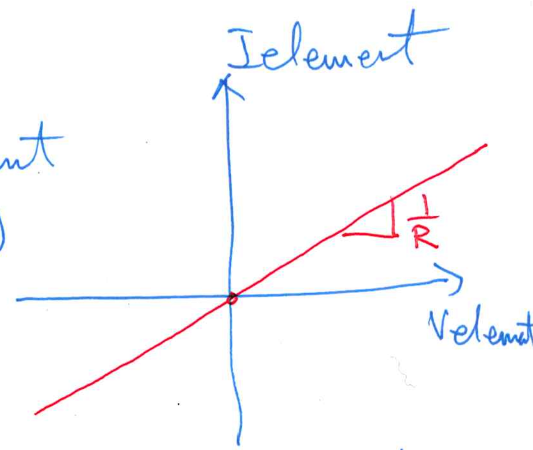



## 2. Resistor

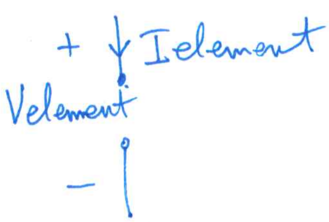



$V_{element} = R \cdot I_{element}$

⇕ (Ohm's law)

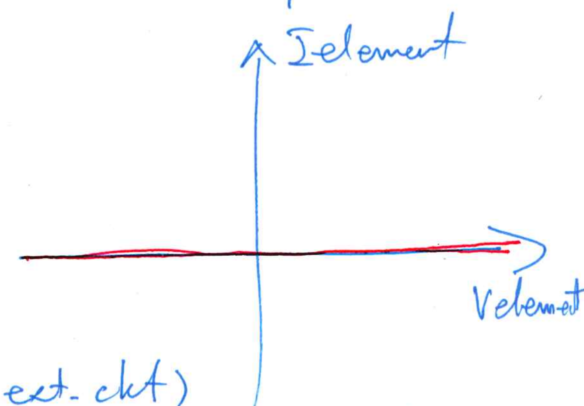



## 3. "Open" circuit

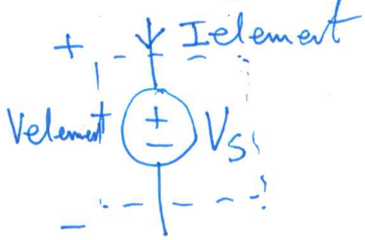



$I_{element} = 0$

$V_{element} = ?$
(set by ext. ckt)

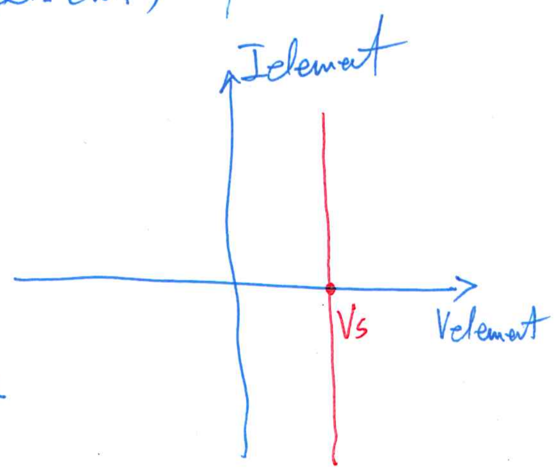



## 4. Voltage source

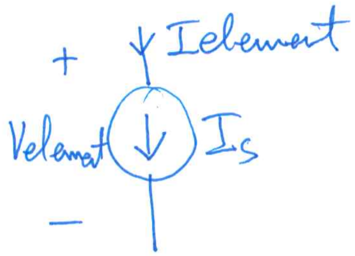



$V_{element} = V_s$

$I_{element} = ?$
(set by ext. ckt)

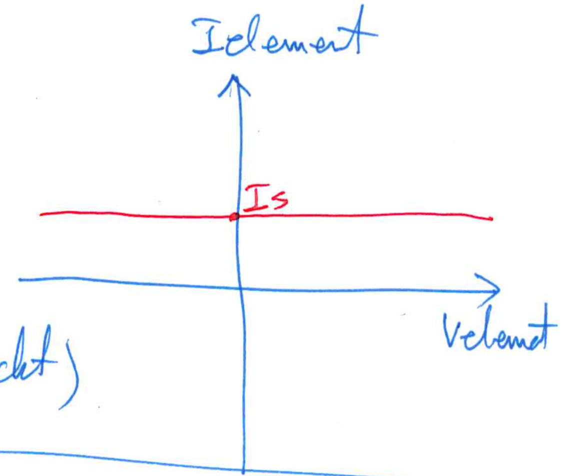


(l3) (5) Current source




$I_{element} = I_s$

$V_{element} = ?$ (set by ext. ckt)

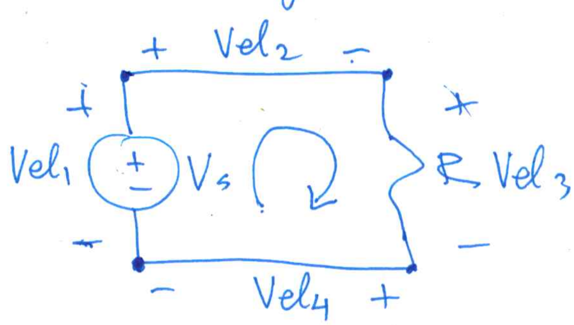



**Velement and Ielement can be positive or negative.**

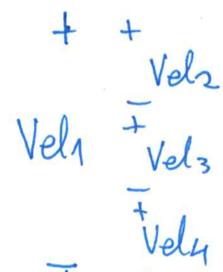
## Rules for circuit analysis:

(1) KVL : sum of voltages across the elements in a loop = 0

example:



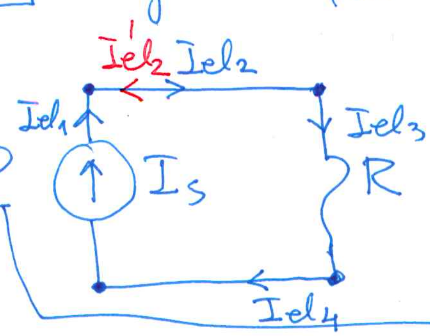

$V_{el1} - V_{el2} - V_{el3} - V_{el4} = 0$




(2) KCL : The current flowing into any junction **must** equal the current flowing out

example:

$I_{el1} + I_{el2}' = 0$




$I_{el1} = I_{el2}$

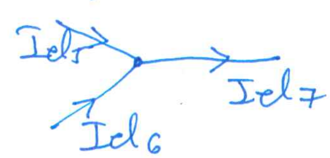
$I_{el2} = I_{el3}$

$I_{el3} = I_{el4}$

~~$I_{el4} = I_{el1}$~~

$I_{el2} - I_{el1} = 0$

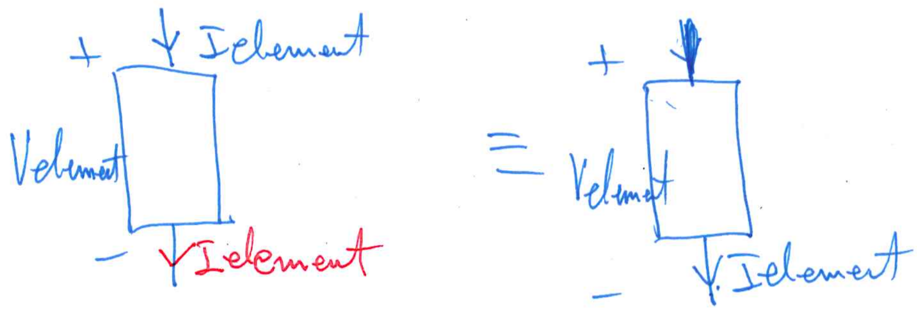
example 2:




$I_{el5} + I_{el6} = I_{el7}$

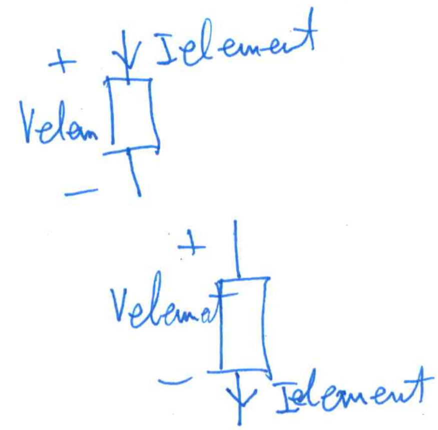
(L4)

## KCL within the element




## Passive sign convention

$I_{element}$ goes into a +
or out of a −

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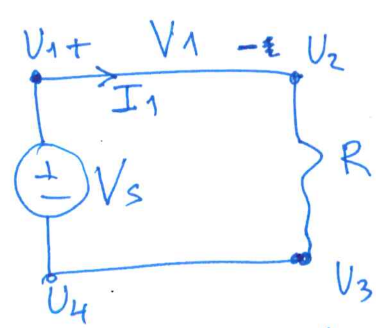
## Circuit analysis algorithm:

voltage = difference of two potentials

Find: currents through elements and potentials of inputs / outputs of each element (junctions)



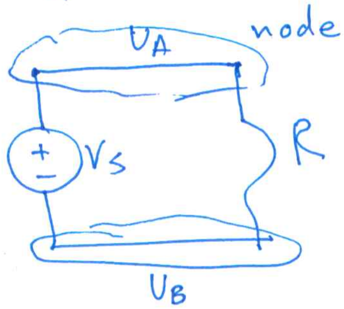

$U_1, U_2$ potentials

$\rightarrow V_1 = U_1 - U_2$ (voltage)

$I_1$ (current)

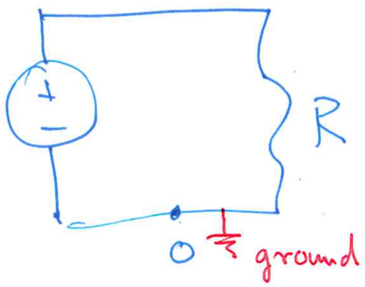
$V_1 = 0$ | $= U_1 - U_2 \Rightarrow U_1 = U_2$

wire element def.

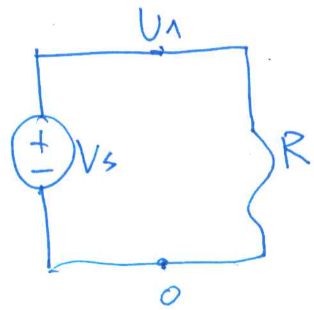



← collapse junctions with same potential into a node.

**Step 1:** Pick a "refence" node and label it as a "0" potential. All voltages measured relative to this node.

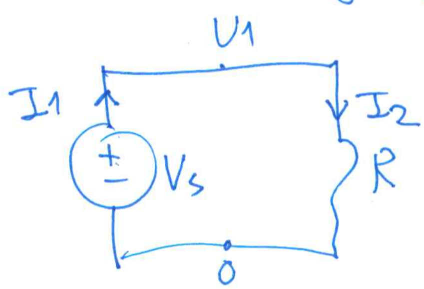



**Step 2:** Label all remaining nodes as potentials $V_i$ $[V_1 \ldots V_{x-1}]$

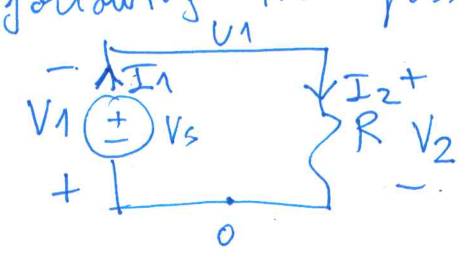



$V_1 - 0 = V_1$ (voltage between node 1 and node "0")

**Step 3:** Label all branch currents with $I_m$. Arbitrarily pick directions of $I_m$. $[I_1 \ldots I_k]$




**Step 4:** Add ± element voltages to each element following the passive sign convention

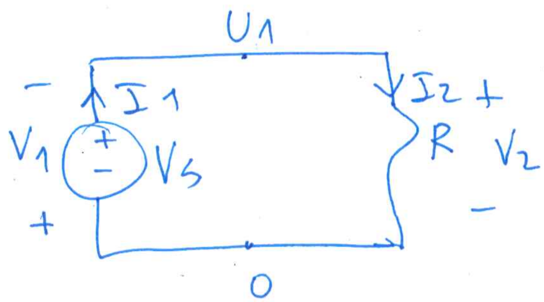



(independent of what is in the element)

**step 5:** $A\vec{x} = \vec{b}$

$$\vec{x} = \begin{bmatrix} I_1 \\ \vdots \\ I_K \\ U_1 \\ \vdots \\ U_{N-1} \end{bmatrix}$$

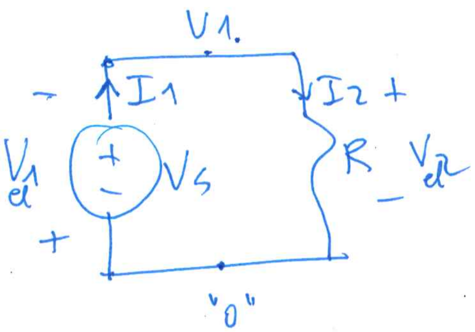
**step 6:** Use KCL to fill as many rows of A as possible (lin. indep) #Nodes − 1 = N − 1




$I_1 = I_2 \Rightarrow I_1 - I_2 = 0$

$$\begin{bmatrix} 1 & -1 & 0 \\ & & \\ & & \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ U_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \end{bmatrix}$$

**step 7:** Use current-voltage relationships for each element to fill the rest of the A matrix




Voltage source: $V_{el1} = -V_S$

$V_{el1} = 0 - U_1 = -U_1$

$\underline{U_1 = V_S}$ ✓

Resistor:

$$\left.\begin{aligned} V_{el2} &= I_2 \cdot R \\ V_{el2} &= U_1 - 0 = U_1 \end{aligned}\right\} \Rightarrow U_1 = I_2 R \Rightarrow I_2 R - U_1 = 0$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & R & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ U_1 \end{bmatrix} = \begin{bmatrix} 0 \\ V_S \\ 0 \end{bmatrix} \Rightarrow \boxed{\begin{aligned} U_1 &= V_S \\ I_1 &= \frac{V_S}{R} \\ I_2 &= \frac{V_S}{R} \end{aligned}}$$